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A realtime adaptive system for dynamics recognition

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Abstract

We propose and investigate an adaptive system for the recognition of the dynamics of an external time series. The system consists of a pool of internal dynamical elements, each of which represents a specific dynamical type. Each of the elements is forced by the external time series to the latter dynamics. We use the absolute value of the control signal, i.e., the strength of forcing, as a criterion for which of the elements out of the pool fits best to the dynamics of the time series. By means of an averaging process the system is able to create a new dynamical element as a "mirror system" of the external one. This adaptation can be performed continuously and impressively quickly even if the dynamical type of the external signal undergoes sudden qualitative changes. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Guided by the idea that the brain is not only a perceptive apparatus but also has the capability to simulate (stimulus and simulus) [1–3] we propose an adaptive network for the recognition of external dynamics. The internal representation thereby is able to simulate and thus has predictive potential very much like we are able to estimate whether we can safely cross the street when a car approaches. However, quite frequently we have to react to sudden changes in the external dynamics. The alternation between "stimulus" and "simulus" leads to a continuous update of the internal representation of the dynamics which in turn can be used for simulation.

In the following we propose a system which allows for an adaptation "on the fly" as mentioned above. Since it seems to be very unlikely that nature uses complicated regression algorithms we have to search for a plausible and easily implementable mechanism. We suggest to use a mechanism that is based on the chaos control method by Pyragas [3]. Some experimental results [4] can be interpreted as evidence that chaos control and especially Pyragas' method may play an important role in brain dynamics [5–7]. Furthermore, it has been shown that Pyragas' method can be derived from diffusive coupling of chemical reactors in a limit [8] which is a further indication for its potential relevance of modeling natural processes.

2. Brief recapitulation of Pyragas' control method

Assume $\mathbf{x}$ and $\mathbf{x}'$ to be the states of two dynamical systems of the same dimension n and the same dynamics $\mathbf{f}$ which are given by the differential equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x};\beta), \qquad \beta = (\beta_1, \beta_2, \ldots, \beta_m), \\ \dot{\mathbf{x}}' &= \mathbf{f}(\mathbf{x}';\beta'), \quad \beta' = (\beta'_1, \beta'_2, \ldots, \beta'_m), \end{aligned} \tag{1}$$

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where β and β' are sets of fixed parameters. Assume further that one of the parameters of the unprimed system is different from the corresponding one in the primed system, $\beta_k \neq \beta'_k$, and the other ones are equal $\beta_i = \beta'_i, i \neq k$. If now the difference of at least one pair of corresponding variables (say the first) multiplied by a suitable chosen factor K is added to the unprimed system

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \ldots, x_n; \beta) + K(x'_1 - x_1), \\ \dot{x}_2 &= f_2(x_1, x_2, \ldots, x_n; \beta), \\ &\vdots \\ \dot{x}_n &= f_n(x_1, x_2, \ldots, x_n; \beta), \end{aligned} \tag{2}$$

this unprimed system will be forced to the dynamics of the primed controlling system, at least if the difference of the dynamics is not too extreme. If the difference of the system parameters is relatively small, the value of the control term $K(x'_1 - x_1)$ will be negligible on the long term. The usual application of Pyragas' method is to stabilize previously unstable periodic orbits in nonlinear systems in the chaotic regime [9]. In the following we slightly deviate from the original application. We refrain from being able to stabilize with an almost vanishing control term as is the case in the aforementioned application of chaos control.

3. Introduction of the adaptive system

In the first approach we use the time series of the x-variable of Rössler's system [10]

$$\begin{aligned} \dot{x}_E &= -y_E - z_E, \\ \dot{y}_E &= x_E + 0.2 y_E, \\ \dot{z}_E &= 0.2 + x_E z_E - \alpha_E z_E, \end{aligned} \tag{3}$$

as a given external signal. The subscript E refers to "external". The parameter α_E is fixed at 5.7 in a first step which leads to the time series of Fig. 1.

Now we choose six further Rössler systems with parameters

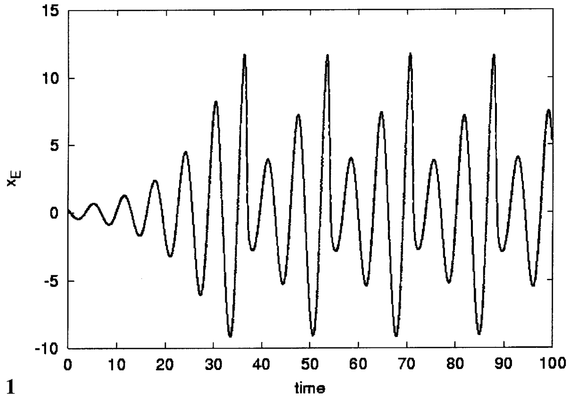
$$\begin{aligned} &\alpha_1 = 5.64, \quad \alpha_2 = 5.66, \quad \alpha_3 = 5.68, \\ &\alpha_4 = 5.72, \quad \alpha_5 = 5.74, \quad \alpha_6 = 5.76, \end{aligned} \tag{4}$$

to constitute a pool of internally given dynamical types. Each of these internal dynamics is forced to the external time series by means of

$$\begin{aligned} \dot{x}_i &= -y_i - z_i + K(x_E - x_i), \\ \dot{y}_i &= x_i + 0.2 y_i, \\ \dot{z}_i &= 0.2 + x_i z_i - \alpha_i z_i; \quad i = 1, 2, \ldots, 6, \end{aligned} \tag{5}$$

where K is a coupling constant which is chosen to be 1 in the following. At each time step ($h = 0.01$ throughout the paper) of the integration of the differential equation we compute the minimum of the absolute value of the six coupling terms. Fig. 2 shows the index (corresponding to the subscripts used in Eq. (4)) of the dynamics that has the smallest control term versus time. The initial conditions of System 1 have been chosen equal to those of the external system which explains that System 1 has the smallest control term in the beginning. In addition, the external system itself has a transient phase in the beginning of about 20 time units duration as can be seen in the time series of Fig. 1. However, after the transient time of 20 units the smallest control term alternates between Systems 3 and 4 which is plausible since the parameter value of the external system lies in-between the parameter values of that pair of internal systems. Fluctuations cause a negligible amount of events where the minima are found in the other systems. These rare events can be easily suppressed by using a moving average of the absolute value of the forcing term over, say three, time steps.

One can use the "switched-on" dynamics, i.e. that one with the temporarily smallest control term, to reconstruct the attractor of the external Rössler system by using the temporarily produced trajectory pieces, for example. However, since our goal is to construct an adaptive system with simulation capability we instead create a new element within the internal pool of dynamics that mimics as precisely as possible the external dynamics.



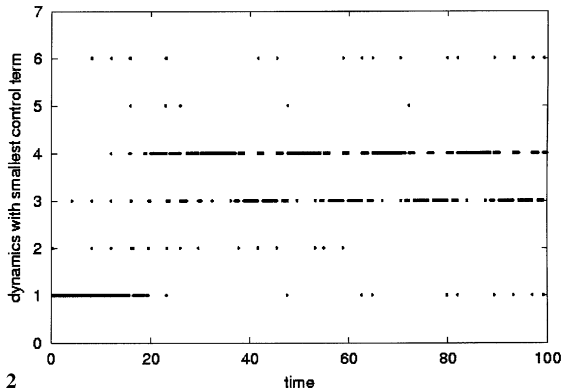


Fig. 1. The externally presented time series computed from the x variable, x_E, of the Rössler system of Eq. (3).

Fig. 2. Switched-on dynamics by time. After a transient phase the internal representation of the external dynamics switches between two dynamics that are closest to the external one in terms of the difference of the parameter values.

4. Creation of a mirror dynamics

To create a new element within the pool of internal dynamics as a mirror of the external dynamics we use the relative frequency p_i of the switched-on dynamics to compute a continuous update of the parameter value of the new system through a linear combination given by

$$\alpha_M(t+h) = \sum_{i=1}^{6} p_i(t)\alpha_i(t) + p_M(t)\alpha_M(t), \qquad \text{with } p_M(t) + \sum_{i=1}^{6} p_i(t) = 1. \tag{6}$$

Thereby, M refers to the mirror system with parameter α_M. The mirror system itself participates at the "competition" and is thus able to confirm its own parameter value α_M if it has a high switch-on probability p_M.

Since it is neither practicable nor plausible in the sense of a "natural" application to use the whole history we estimate p_i and p_M, respectively, by means of a moving average over the recent past, say 10 time steps which corresponds to a time interval of length 0.1. Fig. 3(a) shows the temporal behavior of the new parameter α_M compared to the constant parameter α_E of the external system. We observe a quick adaptation, however, with a relatively large remaining fluctuation between the neighbored parameter values. This fluctuation can be easily explained. A large change in the the value of α_M unsynchronises the mirror system and the external one for a while so that the neighbored systems have smaller control terms within that period which in turn de-adjusts α_M. This can be avoided by limiting the changes of α_M. If one truncates the value given by Eq. (6) to $\text{sign}(\alpha_M) \times \min\{0.0002, \text{abs}(\alpha_M)\}$, i.e., allows a maximum change of 0.0002 amplitude, this leads to a highly significant reduction of the fluctuation as can be seen in Fig. 3(b). Another

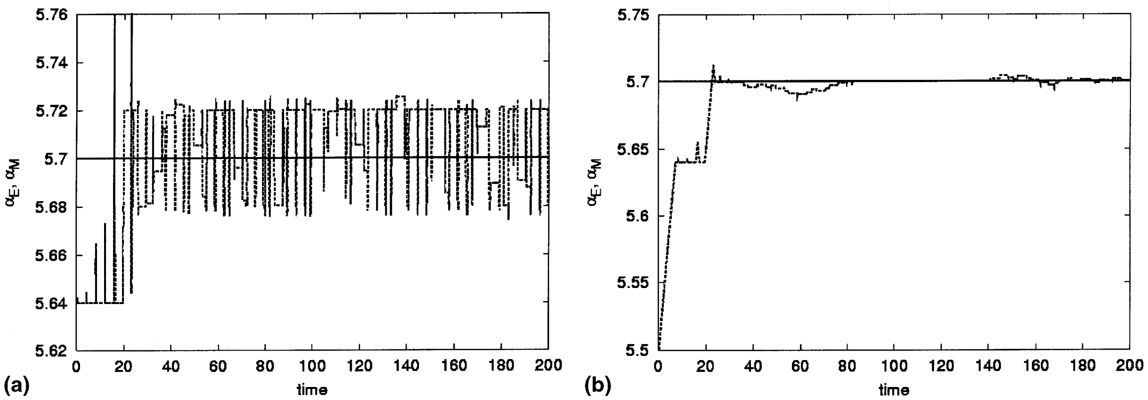


Fig. 3. (a) The parameter adaptation of the mirror system using Eq. (6) leads to an unsatisfactory fluctuation. (b) The fluctuation can be reduced highly significantly by limiting the parameter changes.

possible corrective would be to use a probability distribution that prefers to retain the given value. But anyway, our result so far is quite satisfactory. We postpone the refinements to a forthcoming paper.

Our next step is to investigate another basic feature of the adaptive system, namely the question what happens when the parameter value of external Rössler system undergoes sudden changes.

5. Sudden qualitative changes in the external time series

For our next investigation we stick with the pool of six internally given Rössler systems characterized by the set of parameters of Eq. (4). For the moment we refrain from creating a mirror dynamics and rather look what happens with the switch-probabilities when the external dynamics undergoes qualitative changes. These changes are realized by a stepwise time course of the external parameter α_E according to

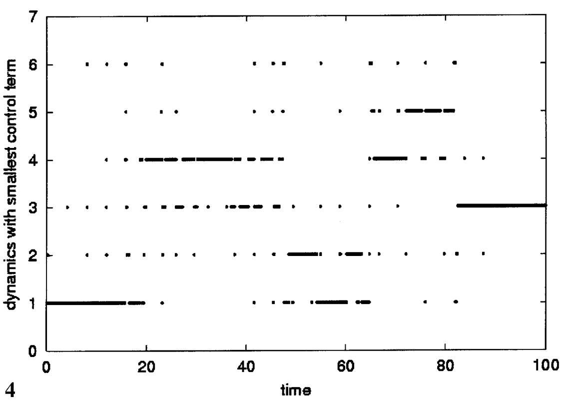
$$\alpha_E(t) = \begin{cases} 5.70, & 0 \leqslant t < 40, \\ 5.65, & 40 \leqslant t < 60, \\ 5.73, & 60 \leqslant t < 80, \\ 5.68, & t \geqslant 80. \end{cases} \tag{7}$$

Fig. 4 shows the result. One sees that it roughly takes 5 time units to "find" the neighbored or, in the last step at $t = 80$, the internally exactly represented dynamics. To get an idea what this means please confer Fig. 1. One sees that five time units roughly correspond to one cycle of the external time series. In other words, the synchronization is realized very quickly.

Let us now re-introduce the adaptive mirror system. We stick with the adaptation procedure introduced above with the 0.0002-threshold for the magnitude of the changes of α_M. We extend the time behavior of the external parameter α_E to

$$\alpha_E(t) = \begin{cases} 5.70, & 0 \leqslant t < 40, \\ 5.65, & 40 \leqslant t < 60, \\ 5.73, & 60 \leqslant t < 80, \\ 5.68, & 80 \leqslant t < 100, \\ 5.74, & 100 \leqslant t < 120, \\ 5.67, & t \geqslant 120. \end{cases} \tag{8}$$

The parameter of the mirror system has an initial value of $\alpha_M = 5.5$. The resulting time behavior of α_M can be seen in Fig. 5. By noting that sudden changes in α_E lead to transient phases in the external dynamics the adaptive behavior again is very satisfactory. We observe a quick adaptation in the beginning as well as during the time course. Towards the end, we see a slight overestimation of the parameter. This tendency can already be seen in Fig. 2 where the alternation between the two (with respect to α) symmetrically neighbored systems slightly prefers the upper System 4. This implies some refinement in future work to tackle this nonlinear effect.



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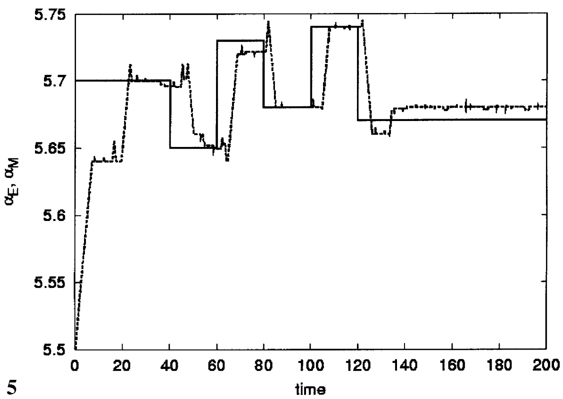

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Fig. 4. Switched-on dynamics by time. The parameter α_E of the external system undergoes stepwise changes at $t = 0, t = 40, t = 60, t = 80$ according to Eq. (7).

Fig. 5. Parameter adaptation in the mirror system as a response to sudden changes in the parameter value of the external system.

6. Adaptation of two parameters

We once more extend our adapative system, namely to a two-parameter adaptation. We compute the external time series from the x-variable of the following Rössler system,

$$\begin{aligned} \dot{x}_E &= -y_E - z_E, \\ \dot{y}_E &= x_E + \beta_E y_E, \\ \dot{z}_E &= 0.2 + x_E z_E - \alpha_E z_E, \end{aligned} \tag{9}$$

where we have introduced the additional parameter β_E chosen to be 0.2 in the following investigation. The internal system now consists of an array of 6×6 elements

$$\begin{aligned} \dot{x}_{ij} &= -y_{ij} - z_{ij} + K(x_E - x_{ij}), \\ \dot{y}_{ij} &= x_{ij} + \beta_j y_{ij}, \\ \dot{z}_{ij} &= 0.2 + x_{ij} z_{ij} - \alpha_i z_{ij}; \quad i = 1, 2, \ldots, 6; \quad j = 1, 2, \ldots, 6, \end{aligned} \tag{10}$$

where we stick with the set of six values for α_i as given by Eq. (4) and choose the six values for β_j to be

$$\begin{aligned} &\beta_1 = 0.15, \quad \beta_2 = 0.17, \quad \beta_3 = 0.19, \\ &\beta_4 = 0.21, \quad \beta_5 = 0.23, \quad \beta_6 = 0.25. \end{aligned} \tag{11}$$

Again we construct a mirror system by means of a parameter adaptation similarly to the above described method. To this end we sum up over the seven forcing terms

$$S_i = \mathrm{abs}(x_E - x_M) + \sum_{j=1}^{6} \mathrm{abs}(x_E - x_{ij}) \tag{12}$$

and use the minimum of S_i for the computation of a relative frequency for α_i within the past 10 time steps and analogous for the other parameter values. This is a very simple and somewhat coarse method, however, it works quite well as you can see in Fig. 6 where the time courses of α_M (6(a)) and β_M (6(b)), respectively, are shown. Again, the subscript M stands for the mirror system that itself participates at the competition. The initial value for α_M has been chosen to be 5.5 and that one for β_M to be 0.24. The external parameter β_M has been kept fixed, however, α_M has undergone stepwise changes given by

$$\alpha_E(t) = \begin{cases} 5.70, & 0 \leqslant t < 60, \\ 5.65, & 60 \leqslant t < 120, \\ 5.74, & t \geqslant 120. \end{cases} \tag{13}$$

Again, as in the one-parameter case above, we used a threshold for the momentary changes in the parameter values, namely 0.0002 for the changes in α_M and 0.0004 for the changes in β_M, respectively.

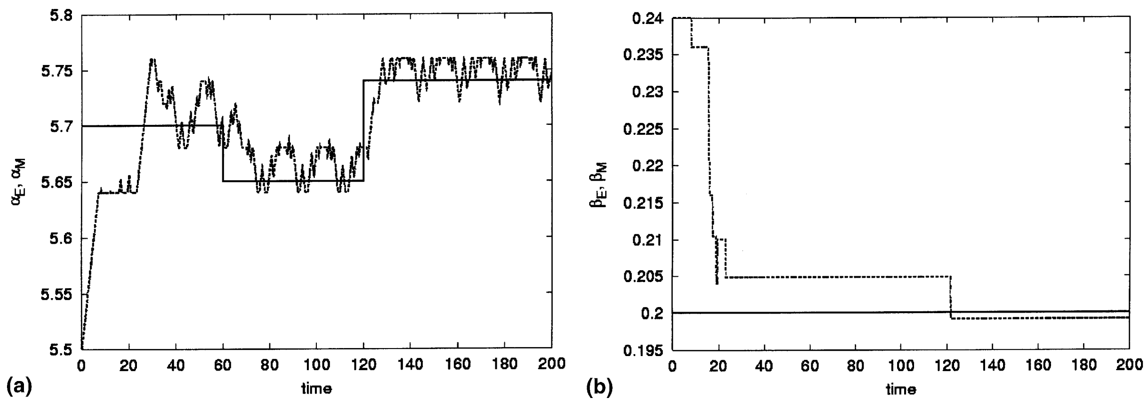

Fig. 6. Two-parameter adaptation in the mirror system as a response to sudden changes in one of the parameter values of the external system: (a) adaptation of the first parameter that undergoes sudden changes and (b) adaptation of the second parameter.

The differences to the one-parameter case are obvious. The frequency of changes in the external dynamics had to be lowered since the adaptation is done a bit slower. Also, the retaining fluctuation of α_M is larger. In contrast to α, the other parameter β seems to be slightly systematically underestimated. We already mentioned this nonlinear effect with respect to α. However, recalling the simplicity of the method, we still have a very convincing result which surely can be improved by gradual refinements in future work.

7. Discussion

The proposed system for dynamics recognition contains an element, called mirror system, that mimics an external dynamics after parameter adaptation and is thus capable to simulate. Of course, if the external dynamics is already represented within the pool of internal dynamics the adaptation is executed very quickly. Therefore, it would be advantageous to have a memory at least for frequently presented external dynamics. To introduce memory without blowing up the dimensionality of the pool of internal dynamics one can choose an existing element out of the pool that undergoes the adaptation, namely one that has not been used for a long time and at the same time is not too different from the external dynamics. Very much like in other natural processes we thereby introduce memory with a certain loss rate.

A further task for future work is to extend the system to different types of dynamics. For example, one may add some elements to the pool of internal dynamics that correspond to the Lorenz system. Almost no doubt that this system will recognize the correct dynamics if either a Rössler or a Lorenz dynamics is presented externally. It is an interesting question how the system reacts to a cross dynamics of Rössler plus Lorenz type. For example, if a sum signal

$$x(t) = \gamma x_1^{\text{Rössler}}(t) + (1-\gamma)x_1^{\text{Lorenz}}(t) \tag{14}$$

is presented to the adaptive system, one can vary γ in the range between 0 and 1 to switch on "Rössler" or "Lorenz" or a cross type. This leads to the idea that for a flexible application one has to search for dynamical elements within the pool that allow for the creation of a large range of different dynamical types. The adaptation, therefore, has to be extended from pure parameter adaptation to recognition of basic dynamical elements that have to be combined. We are very confident that the proposed system has the potential to proceed in this direction.

Last but not the least, we expect a variety of technical and biomedical applications. Assume, for example, a patient suffering from chronical heart arrhythmias that urgently necessitates immediate therapeutical intervention when it suddenly appears. In this case a rather well-working model dynamics exists with which a pool of internal dynamics for an adaptive system can be created. This example may suffice to emphasize the potential relevance of the proposed realtime adaptive system.

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