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Usability of synchronization for cognitive modeling

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Abstract

We discuss the synchronization features of a previously introduced adaptive system for dynamics recognition in more detail. We investigate the usability of synchronization for modeling and parameter estimations. It is pointed out inhowfar the adaptive system based on synchronization can become a powerful tool in modeling. The adaptive system can store modules of pre-adapted dynamics and is potentially capable of undergoing self-modification. We compare the stored modules with pre-knowledge that a modeler puts into his or her models. In this sense the adaptive system functions like an expert system.

In recent time the application potential of synchronization mechanisms of dynamical systems has been recognized and increasingly brought up for discussion (cf. e.g. [1]). The applications cover different fields of control [2], parameter estimation [3] and pattern replication [4]. A link of chaos control mechanisms to brain dynamics has been proposed by Skarda and Freeman [5] and elaborated by others, for a comprehensive review cf. [2].

In this paper we focus on the potentiality of synchronization mechanisms within the fields of life science and cognitive modeling. We point out that synchronization may play an important role in cognitive science and that, therefore, nonlinear science and artificial intelligence studies are closely related. In a recent work [6] we proposed such an adaptive cognitive system that circumvents some of the drawbacks of earlier applications. The most important improvement is a pool of dynamical modules that can be seen as a storage of adapted mirror dynamics which avoid adaptations from scratch. This accounts for two important features of cognitive systems namely the simulation or mirroring capability as discussed in [7,8] and the usage of a priori information.

The paper in hand discusses open problems we encountered in the afore-mentioned work. Specifically we investigate the efficacy of synchronization with respect to the chosen mode of coupling and the dynamical behavior. We therefore briefly recapitulate the underlying basic mechanism. Assume $\mathbf{x}$ and $\mathbf{x}'$ to be the states of two dynamical systems of the same dimension d and the same dynamics $\mathbf{f}$ which are given by the differential equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x};\beta), \quad \beta = (\beta_1, \beta_2 \ldots, \beta_m) \\ \dot{\mathbf{x}}' &= \mathbf{f}(\mathbf{x}';\beta'), \quad \beta' = (\beta'_1, \beta'_2 \ldots, \beta'_m) \end{aligned} \tag{1}$$

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where β and β' are sets of m fixed parameters. Assume further that at least one of the parameters of the unprimed system is different from the corresponding one in the primed system, $\beta_k \neq \beta'_k$ (for at least one k). If now the difference of at least one pair of corresponding variables multiplied by a factor K_i is added to the unprimed system,

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, \ldots, x_d; \beta) + K_1(x'_1 - x_1) \\ \dot{x}_2 &= f_2(x_1, x_2, \ldots, x_d; \beta) + K_2(x'_2 - x_2) \\ &\vdots \\ \dot{x}_d &= f_d(x_1, x_2, \ldots, x_d; \beta) + K_d(x'_d - x_d) \end{aligned} \tag{2}$$

$$K_i \geqslant 0 (K_j > 0 \text{ for at least one } j \in \{1, \ldots, d\}),$$

this system will be forced to the dynamics of the primed controlling system. The success of this forcing depends on coupling modes and dynamical features as investigated in the sequel.

Before we proceed with this investigation we briefly outline the scheme of the adaptive system which is depicted in Fig. 1. An external system—experienced through a stimulus—forces internal modules each according to Eq. (2). One of the modules which we call simulus undergoes an adaptation to the external system. The other modules constitute a set of dynamics that more or less resemble the external one and are compared to it. The resemblance is supposed to be reflected in the magnitude of the forcing. The information on the difference or resemblance is used in a superpositional manner to construct a simulus.

For example, assume given a Rossler system as external dynamics represented as time series of at least one variable x'_i then the modules may be defined through a set of Rossler systems differing in the value of a certain parameter. Assume the value of the corresponding external parameter to lie within the span of the internal parameter values. The idea now is to vary the parameter value of the simulus until the forcing term $x'_i - x_i^{\text{simulus}}$ vanishes (for all used components i) which indicates adaptation. This adaptation can in principle been done without using the modules. The magnitudes of the forcing terms $x'_i - x_i^{\text{module}_j}$ of the modules, however, supply additional information and are used as storage of previously experienced dynamics.

Basic parameter estimation procedures have been described in [9,10] where a priori knowledge on the type of external dynamics was used. The usage of modules that span a grid around the external dynamics in our model considerably enhanced the performance of the adaptive system. Additionally, the model allows to abandon a priori knowledge to a certain degree. The modules play the role of previously experienced dynamics that allow a quicker re-adaptation and fine tuning. The a priori knowledge, so to say, is part of the artificial cognitive system itself.

To start the adaptation without any a priori knowledge one may formulate a general n–dimensional model,

$$\dot{x}_i = \beta_{i00} + \sum_{j=1}^{n} \beta_{ij0} x_j + \sum_{j=1}^{n} \sum_{k=1}^{n} \beta_{ijk} x_j x_k + \ldots, \tag{3}$$

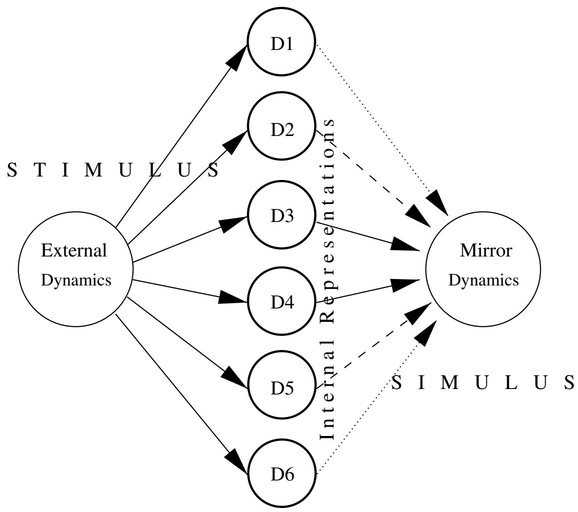

Fig. 1. Cognitive system that adapts a simulus to a 'perceived' stimulus using pre-adapted modules for an efficient adaptation.

up to a necessary but still manageable order (according to Occam's razor). Since the adaptation performance depends on the number of free parameters one can increasingly benefit from pre-adapted modules in the course of time, i.e. from stored a priori knowledge. We mention in passing that a self-modifying feature can be introduced through forcing between the internal modules in combination with appropriate evolutionary criteria of fitness. We expect further improvement with respect to an evolutionary optimization (cf. [11]).

At a first glance it is obvious to use the magnitude of the forcing terms $F_i = |x_i' - x_i^{\text{simulus}}|$ as a criterion for the goodness of synchronization or adaptation, respectively. This surely holds if the F_i almost vanish at every time instant. However, a larger F_i does not necessarily imply that the module does not resemble the target system well due to the fact that some dynamical types are extremely hard to synchronize although they are quite similar. Depending on the mode of forcing even identical dynamics differing only in the initial state may be hard to synchronize. For example, we observed that two Lorenz systems require a relatively large forcing strength K_i in a certain parameter range in order to be synchronized even if the two systems differ only in the initial states. This behavior has to be taken into account when constructing an adaptive system using synchronization mechanisms. In the following we explore this behavior in more detail.

We start by investigating the Rossler system. Assume given a set of modules defined by

$$\begin{aligned} \dot{x}_1^{ij} &= -x_2^{ij} - x_3^{ij} \\ \dot{x}_2^{ij} &= x_1^{ij} + \beta_i x_2^{ij} \\ \dot{x}_3^{ij} &= 0.2 + x_1^{ij} x_3^{ij} - \alpha_j x_3^{ij}, \end{aligned} \tag{4}$$

with $\beta_i = 0.02 + 0.0014i$ and $\alpha_j = 3.0 + 0.025j$; $i,j = 0, \ldots, n-1$; $n = 200$. All $n \times n$ modules are forced according to Eq. (2) by an external system defined through a fixed pair of parameters ($\beta_0 \leqslant \beta' \leqslant \beta_{n-1}$, $\alpha_0 \leqslant \alpha' \leqslant \alpha_{n-1}$) that lies within the parameter range, i.e.,

$$\begin{aligned} \dot{x}_1' &= -x_2' - x_3' \\ \dot{x}_2' &= x_1' + \beta' x_2' \\ \dot{x}_3' &= 0.2 + x_1' x_3' - \alpha' x_3'. \end{aligned} \tag{5}$$

Fig. 2 shows a series of six screenshots that illustrate the evolution of the $n \times n$ forcing magnitudes $F_{ij} = |x_1' - x_1^{ij}|$. The magnitudes F_{ij} are depicted as grey code on the {i, j}-plane, whereby the grey code has been re-scaled for each screenshot. Black areas code for almost vanishing F_{ij}s and white areas indicate large differences between external system and module. The parameter of the external system is marked as a filled white circle.

The simulation has been started with zero forcing ($K_i = 0$ for all three components i) but with identical initial states of all modules and the external system. Therefore, all $F_{ij} = 0$ for $t = 0$. The first screenshot depicts the F_{ij}s after a small time duration $t = 148$ (iteration steps) where emerging differences can be observed indicated through the bright areas. After a further time duration (at $t = 1433$) the chaotic regime can be clearly recognized by a heavily fluctuating area emerging in the upper right corner indicating the absence of phase correlations between the neighbored modules. In other areas, however, one observes standing wave-like oscillations indicating a constant phase relation of the neighbored modules but with different amplitudes of the oscillations. The ''iso-phases'' can be clearly recognized and distinguished from the jittered uncorrelated area. This feature is more pronounced after a further time duration at $t = 4120$ seen in the third screenshot. One can distinct areas of horizontal from areas of curved iso-phases suggesting periodic regimes of similar frequencies that lie outside and, in form of ''islands'', inside the chaotic regime, respectively.

The three screenshots of the second row show the behavior of the F_{ij}s after setting $K_1 = 0.5$ at $t = 4120$. The leftmost screenshot shows an emerging synchronization at $t = 6969$. A remaining small area of non-correlated phases can be observed in the upper part. Please note, that the grey code has been re-scaled. Otherwise a homogeneous grey value would indicate lower differences in the mean compared to the beginning hindering to recognize spatial inhomogeneous behavior which we want to emphasize here. Screenshots five and six have been taken at $t = 9954$ and $t = 17364$, respectively. The fifth image unmasks the upper left corner as a synchronization resistant area in the given forcing mode. Therefore K_1 has been increased to $K_1 = 0.87$ at $t = 9954$ which leads to a stronger synchronization shown in the last image.

The ''live'' visualization reveals some features that are hard to capture with static screenshots. The dynamical behavior of the F_{ij}s shows a characteristic spiral-like wave front moving around the point in the parameter plane that marks the external system. Because of this waxing and waning behavior there is a remaining uncertainty when taking the F_{ij}s as an instantaneous measure for the difference of dynamical systems. However, if one cumulates the F_{ij}s over a time interval, T, according to[1]

[1] We restrict the coupling to the first component.

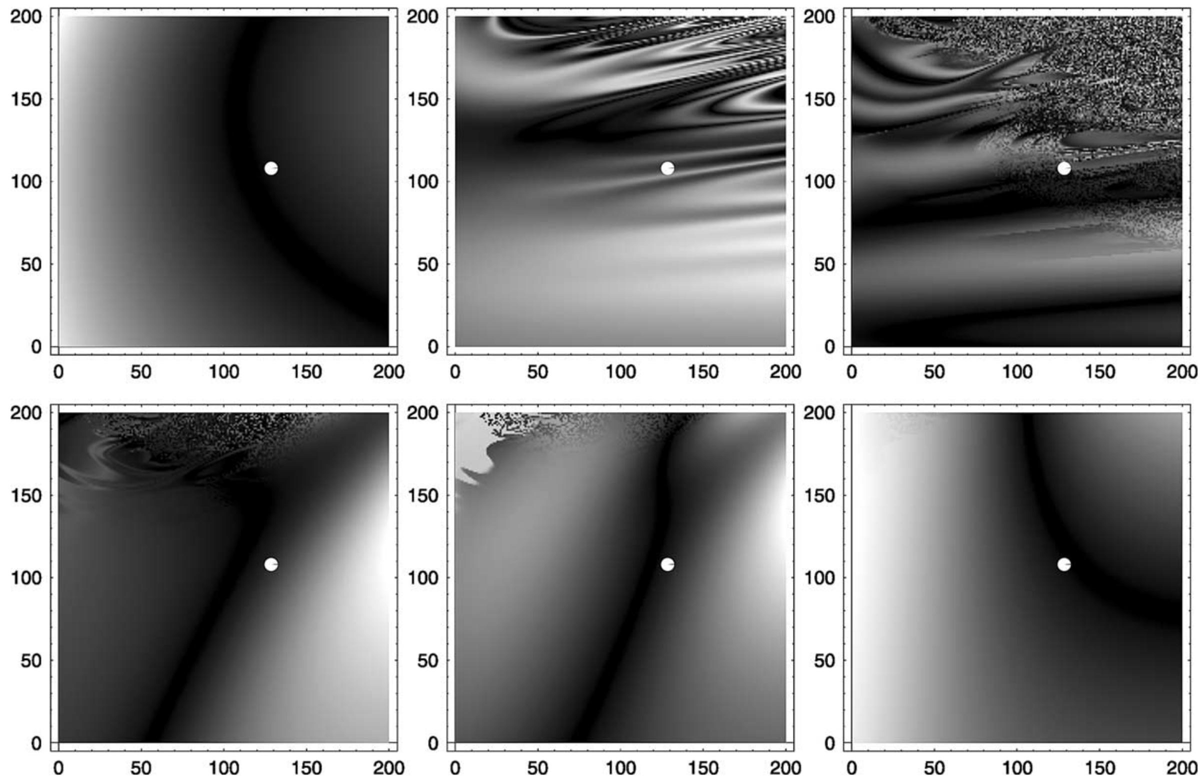

Fig. 2. Synchronization behavior of Rossler's system. The plane of each screenshot shows the increments of the parameter space that define an array of the 200 × 200 controlled systems. The white circle marks the pair of parameters ($\alpha' = 5.7$, $\beta' = 0.2$) that defines the control system. The differences in the first variable between control and controlled systems is depicted as grey code. Cf. text for a detailed description.

$$S_{ij} = \frac{1}{T}\sum_{t=0}^{T-1} |x_1' - x_1^{ij}|, \tag{6}$$

one observes an emerging stationary "potential" surface S_{ij} as depicted in Fig. 3. The function S_{ij} can be interpreted as a likelihood-based function that is to be minimized with respect to the free parameters as pointed out in [9]. If the systems

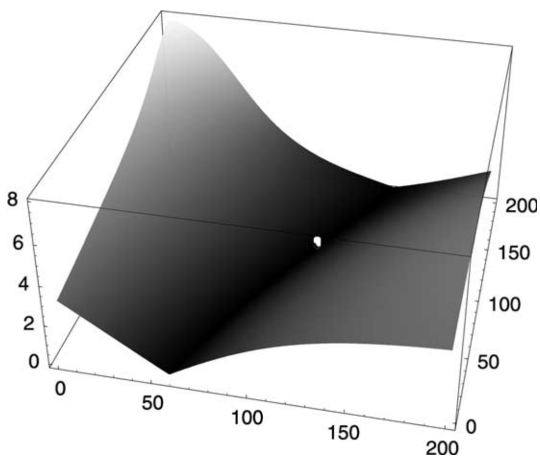

Fig. 3. Cumulated forcing terms shown as a surface function over the parameter space. The defining parameter pair of the control system is shown as a white mark. Cf. text for a detailed description.

do not synchronize the cumulated forcing term of Eq. (6) leads simply to the average distance between the states of control and un-controlled system. With respect to the momentarily given differences F_{ij} the resulting "potential surface" S_{ij} is stationary and smoothed. However, as can be clearly seen in Fig. 3 the potential surface can lead to unpronounced minima with respect to particular parameters, β in the given example.

In a realistic situation one has only a scalar time series available which is expected to represent one variable of a dynamical system. This is the main reason why we use only one coupling variable although we would have all three. The exact number of variables is usually not even known and the measured time series may be a lumped variable of the underlying system. What can be seen from attempts to synchronize the Rossler system in different variables is the fact that coupling in the first and the second one leads to relatively accurate synchronization results whereas synchronization through coupling in the third variable fails. However, we experienced in similar cases that using the modified function $S_{ij} = \frac{1}{T}\sum_{t=0}^{T-1} e^{|x_1' - x_1^{ij}|}$ synchronization is possible. In general, the synchronizability also strongly depends on the number of free parameters and many other settings. This indicates that experienced modelers are often capable of tackling the problem nevertheless, for example by reducing the number of parameters or formulating a modified likelihood as mentioned. In this context we want to emphasize a possible support by a cognitive system. The stored pre-adapted modules of the adaptive system may serve as a kind of expert system that amends the experience of the modeler [12].

The following example demonstrates that it is possible to start the adaptive procedure with little a priori knowledge and without pre-adapted modules. Therefore, we use the maximum likelihood approach to generate a first module by applying it to a modified Rossler system that is relatively hard to synchronize due to a more complex dynamical behavior of the switching variable as can be seen in the left part of Fig. 4. We use the first variable x_1 of

$$\begin{aligned} \dot{x}_1' &= -0.97x_2' - 0.97x_3' \\ \dot{x}_2' &= x_1' + 0.19x_2' - 1.024x_3' \\ \dot{x}_3' &= 0.2 + x_1'x_3' - 5x_3' \end{aligned} \tag{7}$$

as an "observed" time series produced with an integration time step of $\Delta t = 0.08$ and total length of $T = 10000$ steps. As model system that is to be adapted to the time series we choose

$$\begin{aligned} \dot{x}_1 &= \beta_1 + \beta_2 x_1 + \beta_3 x_2 + \beta_4 x_3 + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_2 x_3 \\ \dot{x}_2 &= \beta_8 + \beta_9 x_1 + \beta_{10} x_2 + \beta_{11} x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_2 x_3 \\ \dot{x}_3 &= \beta_{15} + \beta_{16} x_1 + \beta_{17} x_2 + \beta_{18} x_3 + \beta_{19} x_1 x_2 + \beta_{20} x_1 x_3 + \beta_{21} x_2 x_3 \end{aligned} \tag{8}$$

without any a priori information on the 21 free parameters. In other words, we start with zero initial values for all 21 parameters. The initial values of the model variables have been set to the "observed" values. The forcing strength is chosen to be $K_1 = 0.5$. The minimization of $S = \frac{1}{T}\sum_{t=0}^{9999} |x_1'(t) - x_1(t)|$, is executed by using a standard Nelder–Mead minimization procedure. The model output has been computed with a fourth order Runge–Kutta algorithm using a time step of $\Delta t = 0.08$ to mimick the same sampling frequency as the "measured" time series. We repeated the minimization, as usual in minimization procedures with multi-parameter functions, with randomly chosen initial guesses for the parameter values. We stopped the procedure after 100000 runs that did not lead to further improvement of the likelihood.

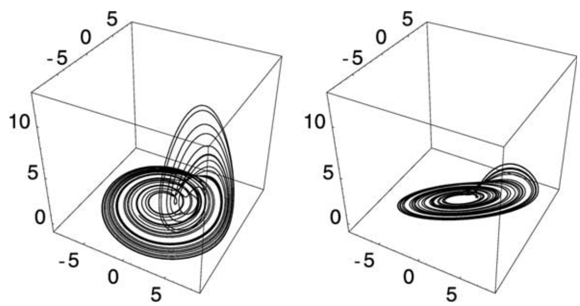

Fig. 4. Phase space of the modified Rossler system of Eq. (7) (left part) and the estimated attractor (right part).

The optimization resulted in the following set of parameter values:

$$\begin{aligned}&\beta_1, \ldots, \beta_7 = 0.90, 1.15, -1.92, -2.31, 0.00, 0.00, 0.00\\&\beta_8, \ldots, \beta_{14} = 0.31, 0.86, -0.92, -0.26, 0.07, -0.29, 0.00\\&\beta_{15}, \ldots, \beta_{21} = 0.29, 0.18, 0.94, -5.05, -0.21, 0.93, 0.01.\end{aligned} \tag{9}$$

The right part of Fig. 4 shows the estimated attractor. It remains to be shown that the estimated attractor is affine to the original one.

To summarize, we presented evidence that synchronization has the potentiality to become a powerful method in the fields of dynamics recognition. The success of modeling by means of synchronization quite frequently depends on subtle modifications by an experienced modeler. There is a lack of both a clear criterion which mode of application of the synchronization procedure will be successful and an absolute criterion of success. For example, in a noisy or distorted time series the synchronization may be "successful" with respect to the distortion. This has to be tested from case to case. We argue that to unfold the power of synchronization new approaches are necessary to tackle this lack of full mathematical analyticity. We investigated a recently proposed cognitive system under this aspect where synchronization reflects a basic mimetic feature in a more general sense as pure parameter estimation. The main idea behind this adaptive system is to mimick to some degree the experience-based modus operandi of experts by bringing in a priori knowledge. We also demand for an opening of scientific principles to a more performative approach that includes simulations and visualizations as a serious tool to gain knowledge. Since the process of understanding obviously takes place in a highly nonlinear system we think that artificial intelligence research and nonlinear science should go hand in hand.

Acknowledgments

We dedicate this article to our friend Mohamed ElNaschie on the occasion of his 60th birthday and express our deep respect towards his unique openness and generosity.

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